Using a Phase Space Statistic to Identify Resonant Objects

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Abstract The identification of resonant objects in radar or sonar, important for object identification, is difficult because existing methods require that the signal have a large signal to noise ratio. I show in this paper that a modified version of the Kaplan Glass (KG) statistic, a phase space statistic used to determine if a signal is deterministic, is sensitive to the properties of resonant objects. The modified KG statistic can be used to detect the presence of a resonant object even when the radar or sonar signal does not come from a deterministic dynamical system. I demonstrate the use of the modified KG statistic both numerically and in a simple experiment.

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In radar or sonar, a transmitted signal is reflected from a target, and the reflected signal is used to find information about the target, such as the distance to the target, or its velocity. Many complex objects have natural resonant modes, and trying to identify the object from these modes is an ongoing problem. The first part of the problem is detecting the modes themselves. A standard approach it to transmit a large impulse. The impulse excites all the resonant modes of the target, and the frequencies of the modes may be determined by analyzing the signal coming from the target after the end of the reflected pulse. Unfortunately, the state excited by the impulse is transient, so the signal loses amplitude with time, resulting in only a small signal to analyze. In addition, other nearby objects will also reflect the impulse and oscillate at their own natural frequencies, adding noise to the already weak transient signal.

It was discovered that a modified version of the Kaplan Glass (KG) phase space statistic is sensitive to the properties of resonant objects. The Kaplan Glass statistic was originally developed to detect whether or not a signal was deterministic, but it was found to be useful for detecting resonance in radar or sonar applications even when the radar or sonar signal was not deterministic. The KG statistic incorporates an average over the phase space, and it may be used with long signals, so it is less sensitive to noise than other resonance detection methods.

Introduction

A common problem in radar and sonar is trying to identify an object from a reflected signal. One common method is to transmit an impulse signal. The impulse excites natural resonances in the target, causing it to ring. After waiting for some time, the decaying signal from the target is recorded and analyzed to determine the resonant frequencies that are present [1].

A disadvantage of this method is that the decaying impulse response is not large, so the signal to noise ratio is low. It is possible to create a larger signal by using a larger impulse, but at some point the transmitter power required becomes excessive.

I have found that a statistic developed by Kaplan and Glass (KG) [2] to determine if a signal came from a deterministic system may also be used to detect if a signal has been scattered by a resonant object. The KG statistic may also be used to determine the time at which a reflected signal returns to the receiver, in the same way that cross correlations are used. Because the KG statistic is essentially linear, and uses averaging over phase space, the KG statistic is robust to noise, so it should be able to detect resonance at a lower signal to noise ratio than the impulse method.

Radar Basics

In the fields of radar and sonar, transmitted signals that are reflected from objects are used to determine the distance to and radial velocity of those objects [3]. Usually, periodic signals or combinations of periodic signals have been used for these applications, but other types of signals such as noise or chaos have also been explored. Chaotic signals may have a large bandwidth, which is useful in increasing the precision with which distances may be measured. To understand this principle, think of a pulse or a wave packet: the more the wave packet is localized in time, the greater the spread of frequencies used to create the wave packet The application of chaos to radar or sonar has been considered by several groups [4-8]. To measure distance in radar, one actually measures a delay time, so signals with broader bandwidths increase the precision with which distance may be measured.

The Kaplan-Glass Statistic

Given a time series signal $x(t)$, the first step in computing the KG statistic is to embed the signal in a phase space [9]. The phase space dimension and delay may be determined by appropriate methods [10].

If $x(t)$ comes from a deterministic system (and has been properly embedded), then points that are nearby in phase space should have similar dynamics. The KG method constructs a set of dynamics vectors $\delta(t_i) = (x(t_i + \tau_0 + \Delta) - x(t_i + \tau_0), (x(t_i + \tau_1 + \Delta) - x(t_i + \tau_1), \ldots)$ for a set of points that are neighbors in phase space, where $\tau_0, \tau_1, \ldots$ are the phase space delays for each dimension, and Δ is the delay used to compute the derivative vectors. In this paper, Δ is set to 1.

The vectors for the set of neighbors are each normalized to unit vectors, and then they are summed to get the statistic

$$S_{KG} = \frac{1}{N}\sum_{i=1}^{N}\frac{\delta(t_i)}{|\delta(t_i)|} \qquad (1)$$

where N is the number of phase space neighbors in the small region. If all the derivative vectors are parallel, then $S_{KG} = 1$. If the vectors are oriented randomly, then S_{KG} is not 0, but has a finite value that depends on the properties of a random walk. In the original Kaplan and Glass paper, they compute the average displacement of a random walk with the same number of dimensions and points as were used to calculate S_{KG} , and subtract the random walk value from S_{KG} to get a number between 0 and 1, where 0 corresponds to randomly oriented vectors, and 1 corresponds to parallel vectors. In the work presented here, since I am working with short time series, it appears to be more accurate instead to repeat the KG statistic for a group of points randomly selected from the time series *x(t)*. I call this statistic S_R, and subtract it from S_{KG} . The calculation is then repeated for a number of different neighborhoods on the attractor to produce an average statistic $S_A = \langle S_{KG} - S_R \rangle$.

Referenced Kaplan-Glass Statistic for Signal Detection

In radar and sonar, we transmit a signal *x(t)* and receive a signal *y(t)*, which may contain multiple delayed copies of *x(t)*, as well as added noise. If we try to embed *y(t)* in a phase space, we no longer know which points are neighbors in phase space, because their locations have been altered by noise and by the multiple copies of *x(t)*.

To understand how the KG statistic may be used to identify the presence of a delayed signal, begin by assuming that *y(t)* contains only 1 delayed version of *x(t)*, plus added noise. We embed the original undelayed signal *x(t)* and find a set of phase space neighbors on the embedded *x(t)* as before. The neighboring points have indices ($i_0, i_1, \ldots i_N$). We then take a set of points with the same indices in *y(t)*. Because *y(t)* is delayed in time from *x(t)*, this set of points will not in general be neighbors on *y(t)*, so if we compute the KG statistic for this group of points, it will be close to the value for randomly distributed points. The indices of the points used for calculating the KG statistic are based on the reference signal, so I call this version of the KG statistic the Referenced Kaplan-

Glass (RKG) statistic. If we repeat this procedure for many neighborhoods on *x(t)*, the average statistic S_A calculated on *y(t)* will be near 0.

Next, we add a delay *k* to the index of each of the points chosen from *x(t)* and calculate the RKG statistic for the set of points on *y(t)* with indices ($i_0+k, i_1+k, \ldots i_N+k$). If *k* is equal to the time delay of *y(t)* , then the points with these indices will be neighbors in *y(t)*, and the RKG statistic will be large.

The KG statistic was developed to test for deterministic signals, but a low dimensional dynamical system driven by a random signal, or even a broad band signal such as a linear chirp [3], can give a value for the KG statistic similar to the value for a deterministic system. It is known that filtering of a chaotic signal can increase its dimension, if certain conditions on the negative Lyapunov exponents of the filter and the chaotic signal hold [11]. Physically, dimension increase can occur when the filter mixes parts of the chaotic signal that are well separated in time, so that they are uncorrelated. If the frequencies in the chaotic signal are near to the resonant frequency of a resonant system, however, these conditions are unlikely to hold, so dimension increase is not a concern here. The fact that the KG statistic can be large for nondeterministic signals actually makes it more useful for radar and sonar, as it allows a greater variety of signals to be used. Modulated sine waves are common in radar and sonar because they have a constant envelope, which allows them to make more efficient use of transmitter power.

Figure 1 shows the result of using the RKG statistic to detect a delayed signal from a Lorenz system. The original signal *x(t)* is a 2000 point time series of the *x* component of a chaotic Lorenz system [12] , with a time step of 0.05. The signal *y(t)* was the same Lorenz signal delayed by 500 points, with an added Gaussian white noise signal that was 10% of the amplitude of the Lorenz signal.

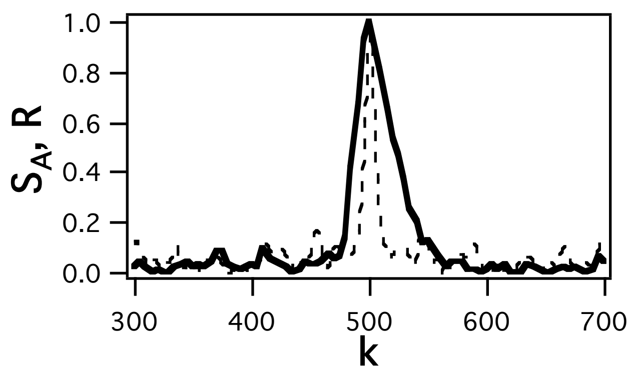

Figure 1. Averaged RKG statistic S_A (solid line) and cross correlation *R* (dotted line) vs delay time step k for a Lorenz signal delayed by 500 points. Both statistics have been normalized so that the largest amplitude is 1.

The *x(t)* signal was embedded in a 3 dimensional phase space, with delays of 0, 5 and 10 points. It was found that signal detection using the RKG statistic was not very sensitive to the specific phase space parameters for the Lorenz signal. Also plotted in Fig. 1 is the cross correlation between *x(t)* and *y(t)*, where

$$R(m) = \sum_{n=0}^{N-m-1} x(n+m)y(n) \qquad (2).$$

Both statistics in Fig. 1 have been normalized so that their largest value is 1. The cross correlation is mathematically equivalent to a matched filter, which is a standard method of signal detection in radar and sonar [3].

It is often necessary to detect a signal in the presence of multiple, overlapping copies of the signal. Figure 2 is a simulation of such a situation. For Fig. 2, the signal *y(t)* contains multiple copies of the Lorenz signal in *x(t)*, with different amplitudes and delays. One of the Lorenz signals has a delay of 200 points and an amplitude multiplier of 1.0, one has a delay of 1000 points and an amplitude multiplier of 0.5, and one has a delay of 1800 points and an amplitude multiplier of 1.0.

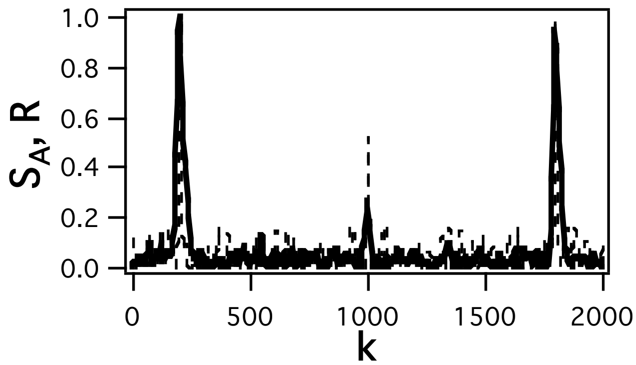

Figure 2. Averaged RKG statistic S_A (solid line) and cross correlation *R* (dotted line) vs. delay time step *k* for a sum of Lorenz signals, with delays of 200, 1000 and 1800 points, and amplitude multipliers of 1.0, 0.5, and 1.0.

The averaged RKG statistic S_A detects all 3 peaks, although the signal to noise ratio for the middle peak is not as good as for the cross correlation.

Signals such as the Lorenz signal are not usually used in radar because they do not have a constant envelope. Constant envelope signals, such as modulated sine waves, make more efficient use of the transmitter power. Figure 3 shows an example of detecting a sine wave that is frequency modulated with random noise. The modulation width of the sine wave, which contained 20 points per cycle, was 10%, and the original signal *x(t)* was embedded in a 3-d phase space, with delays of 0, 5, and 10. The signal *y(t)* contained a copy of *x(t)* delayed by 500 points, with 10% added Gaussian white noise.

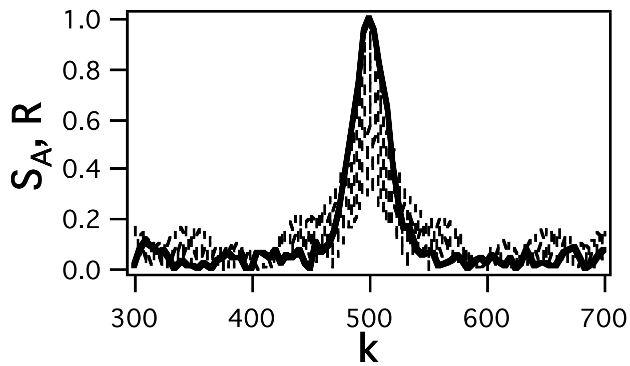

Figure 3. Averaged RKG statistic S_A (solid line) and cross correlation R (dotted line) vs. delay time step k for a noise modulated sine wave signal delayed by 500 points. Both statistics have been normalized so that the largest amplitude is 1.

The cross correlation for the modulated sine wave in Fig. 3 appears to be very complex. The complex curve is caused by a "ripple" in the cross correlation which repeats once every period of the unmodulated sine wave, or every 10 steps. The RKG statistic actually has a slightly narrower central peak than the cross correlation for the modulated sine wave, with the overall signal to noise ratio being about the same.

If the radar or sonar signal is reflected off a moving target, the reflected signal will undergo a Doppler shift, where all frequencies are shifted by an amount proportional to the target velocity. The RKG statistic was tested and was found to be sensitive to a Doppler shift, so it is also useful when reflected signals are Doppler shifted.

Resonance Detection

While it is possible to use the RKG statistic purely for signal detection, there is no advantage to doing so, since it is no better than established techniques. It was found that when a low dimensional damped dynamical system was driven near its resonant frequency, the RKG statistic changed as the driving signal frequency passed through the resonance. This change in the RKG statistic is useful for detecting the presence of a resonance. There are existing techniques for detecting resonance based on exciting the target with an impulse [1], but these methods require a high signal to noise ratio. Resonance detection is used to identify targets based on the natural resonant modes of the target.

For simplicity, I use a sine wave to demonstrate why the RKG statistic is sensitive to a phase shift. Pure sine waves are actually not useful in radar or sonar because they do not give information on location (every cycle is identical, so a sine wave contains no timing information), but it is easiest to explain the statistic with a sine wave.

A sine wave signal $\sigma_1(i) = sin(2\pi i/20)$ with a period of 20 points was generated, where $i = 0, 1, 2, \ldots$. One period of this signal is plotted in Fig. 4. The signal $\sigma_2(i)$ was a copy of $\sigma_1(i)$ whose phase was advanced by 4 points. The signal $\sigma_2(i)$ was also plotted in Fig. 4.

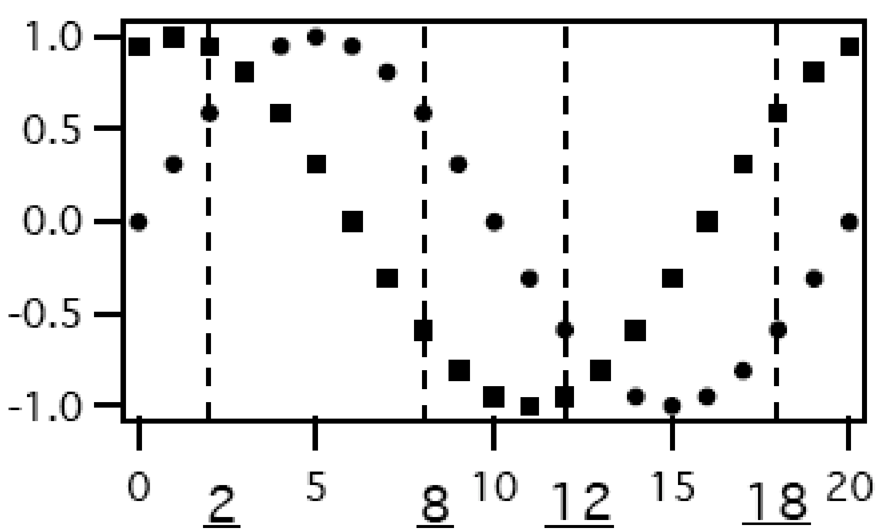

Figure 4. The signals $\sigma_1(i)$ (circles) and $\sigma_2(i)$ (squares). The underlined points 2, 8, 12, and 18 were used for calculating the derivative vectors.

The reference signal $\sigma_1(i)$ was embedded in 2 dimensions, with an embedding delay of half a period (10 points). The embedding delay was chosen to produce an RKG statistic of 0 for a sine wave in phase with $\sigma_1(i)$. The reason that the RKG statistic is zero is that embedding a sine wave with a delay of half of a period collapses the plot onto a line with a slope of -45 degrees. In this plot, points that are reflections of each other about the maximum or the minimum in the sine wave fall on top of each other, so they are nearest neighbors. Because the nearest neighbors are reflected about the maximum or minimum, their derivative vectors will be antiparallel. Figure 5 shows an example where the embedding delay is 9, not 10, in order to spread out the plot so that it is easier to understand.

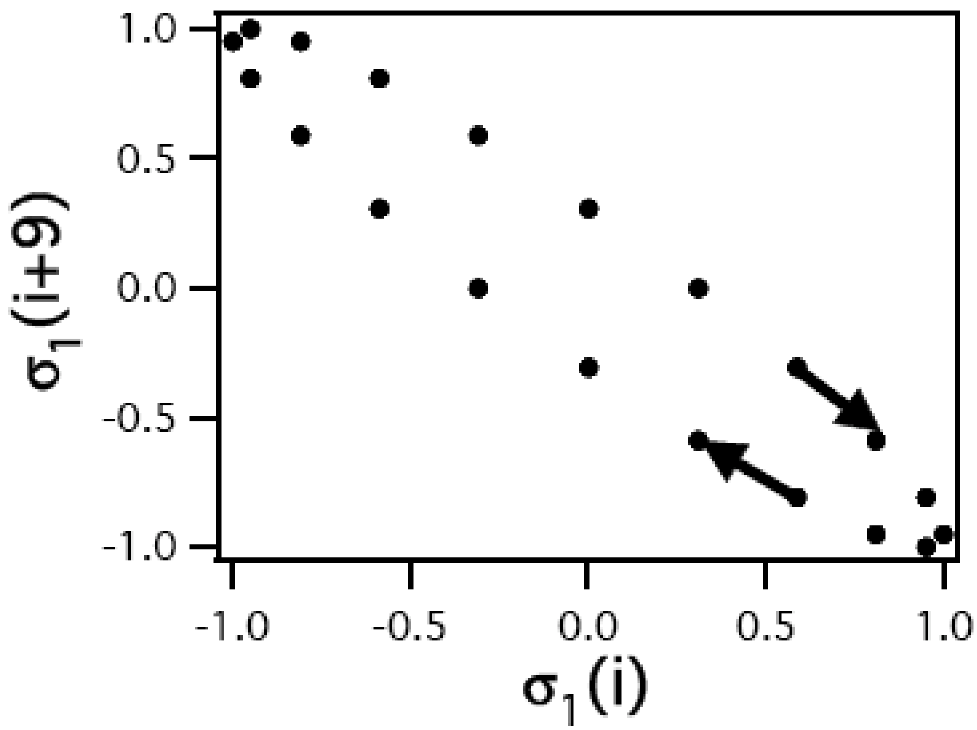

Figure 5. Plot of the reference signal $\sigma_1(i)$ delayed by 9 points vs. the reference signal. The 9 point delay was used to spread out the plot so that it was easier to see. The arrows show the approximate derivative directions for the points (2, 12) and (8, 18). These points fall on top of each other for a delay of 10.

For the example in Fig. 5, the initial index was arbitrarily chosen as 2, so that the first embedded point is (2, 12). The point (8,18) has the same coordinates as (2, 12), as can be seen on Fig. 4. On Fig. 5, the arrows show the direction of the derivatives at points (2, 12) and (8, 18). The derivative vectors are antiparallel.

On Figure 6 is plotted $\sigma_2(i+9)$ vs. $\sigma_2(i)$. Because $\sigma_2(i)$ is phase shifted from $\sigma_1(i)$, the points (2, 12) and (8, 18) are no longer neighbors, and are not reflections of each other about the minimum or maximum of the sine wave.

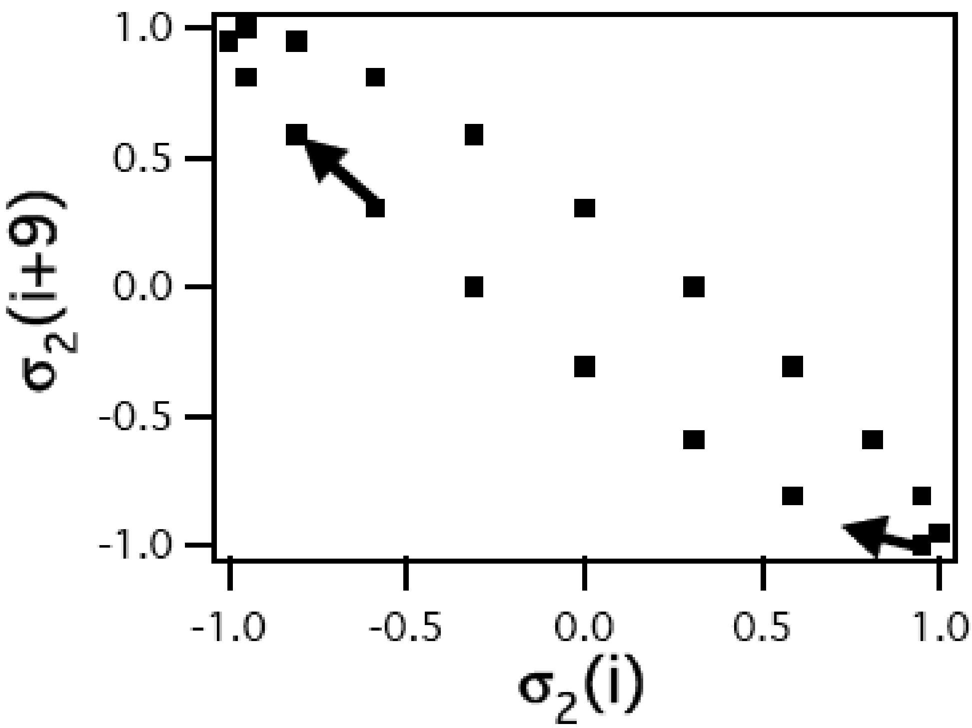

Figure 6. Plot of the signal $\sigma_2(i+9)$ vs. $\sigma_2(i)$. The 9 point delay was used to spread out the plot so that it was easier to see. The arrows show the approximate derivative directions for the points (2, 12) and (8, 18). These points fell on top of each other for the embedded version of $\sigma_1(i)$ a delay of 10.

As can be seen in Fig. 6, the derivative vectors for (2, 12) and (8, 18) are no longer opposite in direction, so they no longer cancel, causing the RKG statistic to have a value > 0.

When a narrow band signal drives a resonant system, the system response goes through a phase shift, with a phase shift of 0 at resonance. If the RKG statistic is applied to the response signal (with the driving signal as a reference signal) with an embedding delay of half a period, then the RKG statistic will be at a minimum at resonance. Different choices of delay are also possible; for example, for a delay of one quarter cycle, points shifted by one cycle of the sine wave will be neighbors, and the RKG statistic will pass through a maximum at resonance.

Numerical Detection of Resonance

A simple damped oscillator was used to numerically simulate an object with 1 resonant mode. The oscillator was described by

$$\begin{aligned}
\frac{d\psi}{dt} &= \alpha_1 \xi \\
\frac{d\xi}{dt} &= \alpha_1 \left(-\gamma\xi - \alpha\psi + s_d \right) \\
\alpha &= 1 \;\; \gamma = 0.1
\end{aligned} \tag{3}$$

where s_d is the signal that drives the resonant system and α_1 can be varied to change the resonant frequency of the resonant system.

Figure 7 shows the detection of the resonance of eq. (3) using the RKG statistic. The driving signal s_d in this case was a sine wave that was frequency modulated by random noise. The output signal ξ from eq. (3) had 10% Gaussian white noise added to it.

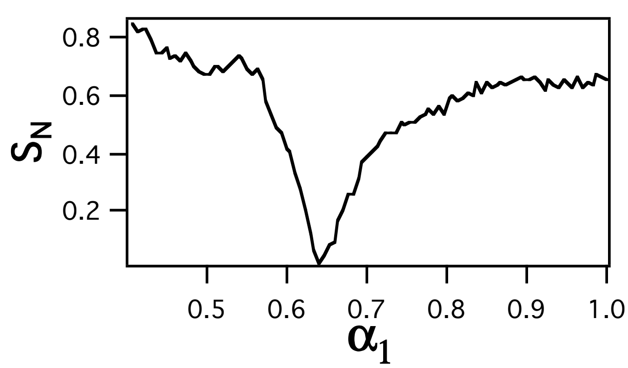

Figure 7. Averaged RKG statistic, normalized by the cross correlation, vs. time constant α_1 for the output signal from the resonant system of eq. (3).

To produce Fig. 7, the time constant of the resonator was swept through resonance (this required less computation than actually sweeping the frequency of s_d). As the time constant passes through resonance, a minimum is seen in the RKG statistic. The averaged RKG statistic was normalized by the amplitude of the cross correlation at the same time delay to correct for any amplitude variation in the signal. The normalized value of S_A is S_N.

Usually in radar or sonar, there are other interfering signals present, such as multiple reflections from multiple targets. Figure 8 shows that the RKG statistic is still useful for resonance detection when a large interfering signal is present. To create Fig. 8, the driving signal s_d was added (with the same time delay) to the resonator output signal ξ. Both signals were normalized to have equal amplitude, and 10% Gaussian white noise was added to the sum of signals. This simulation represents a situation where the resonant signal is obscured by a specular reflection from the target.

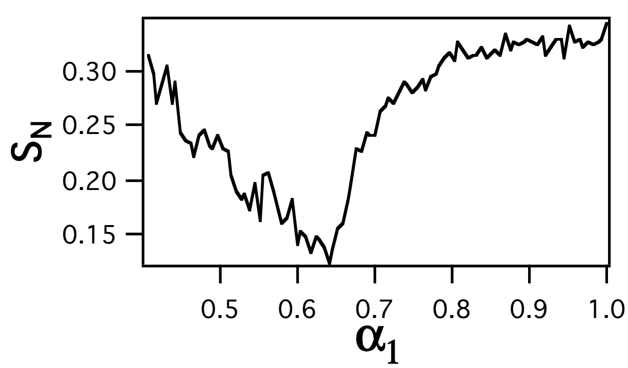

Figure 8. Averaged RKG statistic, normalized by the cross correlation, vs. time constant α_1 for the output signal from the resonant system of eq. (3) when the signal s_d has been added as interference.

It is also possible to detect more complex resonances with the RKG statistic. A set of 3 coupled modes was modeled by

$$
\begin{aligned}
&\frac{d\xi_1}{dt} = \alpha_1\xi_2 \\
&\frac{d\xi_2}{dt} = \alpha_1\left(-\gamma\xi_2 - \alpha\xi_1 - \gamma\left(\xi_2 - \xi_4\right) - \alpha\left(\xi_1 - \xi_3\right) + s_d\right) \\
&\frac{d\xi_3}{dt} = 1.5\alpha_1\xi_4 \\
&\frac{d\xi_4}{dt} = 1.5\alpha_1\left(-\gamma\xi_4 - \alpha\xi_3 - \gamma\left(2\xi_4 - \xi_2 - \xi_6\right) - \alpha\left(2\xi_3 - \xi_1 - \xi_5\right) + s_d\right) \\
&\frac{d\xi_5}{dt} = 0.8\alpha_1\xi_6 \\
&\frac{d\xi_6}{dt} = 0.8\alpha_1\left(-\gamma\xi_6 - \alpha\xi_5 - \gamma\left(\xi_6 - \xi_4\right) - \alpha\left(\xi_5 - \xi_3\right) + s_d\right) \\
&\alpha = 1 \;\; \gamma = 0.1
\end{aligned}
\tag{4}
$$

In this case, the output signal was $\xi_1+\xi_2+\xi_3$, again with 10% added noise. Figure 9 shows the result of sweeping the time constant α_1 for eq. (4) through the resonant region, and calculating the RKG statistic.

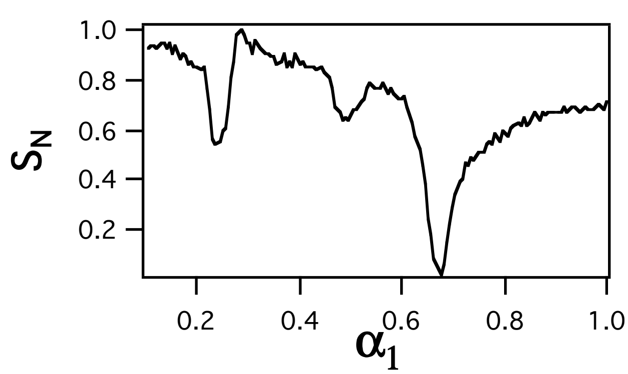

Figure 9. Averaged RKG statistic, normalized by the cross correlation, vs. time constant α_1 for the output signal from the resonant system of eq. (4) showing all 3 resonant modes.

All 3 resonant modes can be seen in Fig. 9.

The RKG statistic could be used to detect resonance when s_d was a chaotic signal, although the reason that detection works is not as obvious as for a simple periodic signal. For Figure 10, a signal from a Lorenz chaotic signal was used to drive the single mode resonant system of eq. (3). Once again, the time constant α_1 was varied, and the ratio of the average RKG statistic S_A to the cross correlation was plotted. The original Lorenz signal was embedded in a 3-d phase space, with delays 0, 11, and 19. The delays were determined by a recently developed method [13].

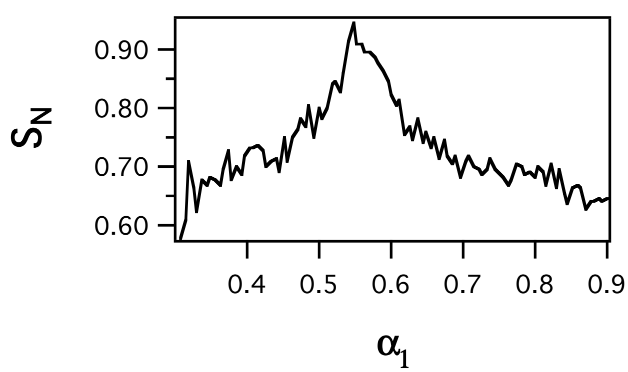

Figure 10. Average RKG statistic S_A, normalized by the cross correlation, when the single mode resonant system of eq. (3) is driven by a Lorenz signal.

The delays used here were closer to a quarter cycle then a half cycle, so the RKG statistic went through a maximum as α_1 was swept through resonance

. The signal to noise ratio for the detection of resonance did vary with embedding parameters, but no comprehensive study of this effect has been done.

Experimental Test

In order to test the RKG statistic in the lab, a small aluminum box was used as an acoustic resonator. The dimensions of the box were approximately 12.7 cm x 10.2 cm x 7.6 cm, and a 3.2 cm hole was drilled in the center of the large face of the box. For the experimental test of the RKG statistic, the box was placed on a stand 61 cm from the speaker. The microphone was near the speaker. Fig. 11 shows the experimental setup.

Figure 11. Diagram of experiment with acoustic resonator (target) to test the RKG statistic.

In the simplest type of sonar experiment to find the distance to the box, the speaker emits a short pulse, which is reflected by the target and detected by the microphone. I conducted this experiment to make sure I could detect the box. The signals used in the pulse experiment are shown in Fig. 12

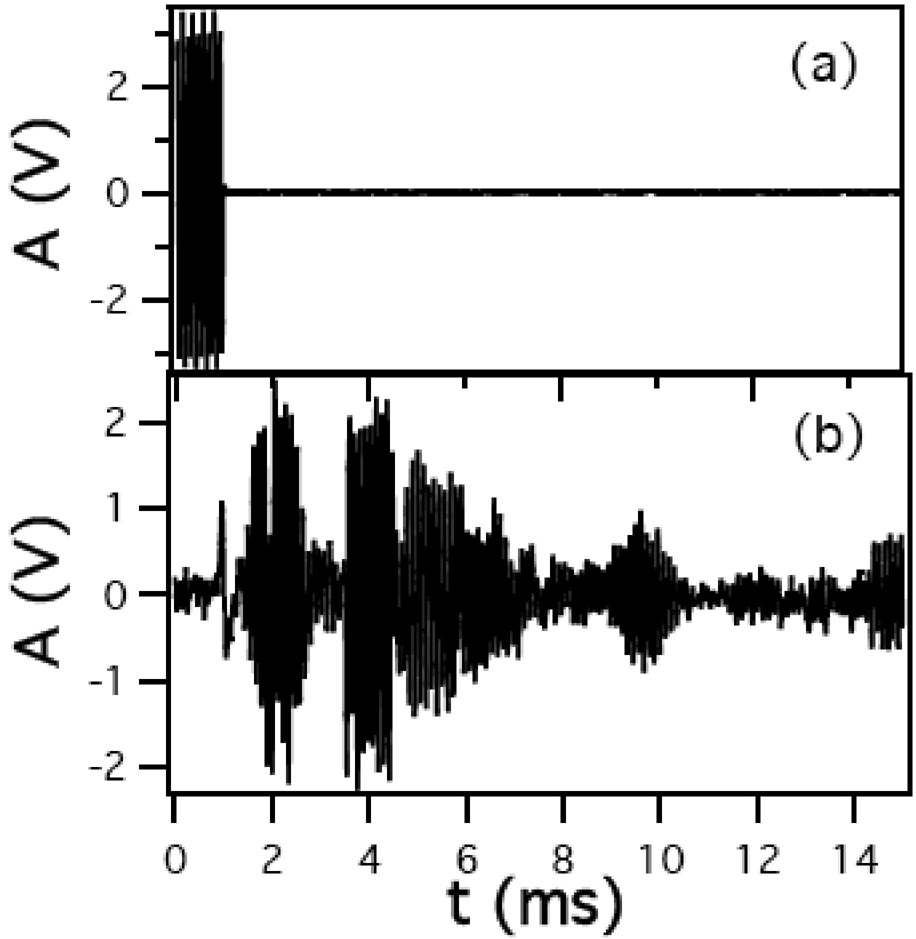

Fig 12. (a) Short 9 kHz pulse emitted from the speaker. (b) Signal detected by the microphone. The first pulse at 2 ms is caused by the signal traveling directly from the speaker to the microphone. The pulse at about 3.8 ms is the signal reflected from the aluminum box. The broad signal after 4 ms is caused by reflections from the tripod that the box sits on, while the signal at 10 ms is a reflection from another object in the room.

The top trace in Fig. 12 shows the actual waveform that was transmitted by the speaker, a 1 ms wide pulse with a frequency of 9 kHz. The bottom trace is the signal actually recorded from the microphone and amplified. At 2 ms, one can see the signal that travels directly from the speaker to the microphone. The reflection from the aluminum box is at about 3.8 ms. The speed of sound in air is about 331 m/s, so the round trip distance from speaker to box to microphone is about 125 cm, and the box is at about 62 cm from the speaker- roughly the same distance as obtained by direct measurement.

The RKG statistic requires a continuous signal, or a long pulse, so the box can't be located simply by looking for a reflected pulse. Instead, the cross correlation between the reflected signal and a reference signal is used. Reflections from different objects will produce peaks in the cross correlation signal at delay times corresponding to the round trip distance between the speaker and the object.

Figure 13 shows the cross correlation signal from the experiment of Fig 11 when the transmitted signal was a 4 kHz sine wave that was frequency modulated with random noise, so that its bandwidth was 3% of its center frequency. The modulated sine wave was produced by an arbitrary waveform generator. The speaker distorted the sine wave badly, so for a reference signal, the direct signal from the speaker to the microphone with no target present was used. A delay of 1.8 ms was subtracted from the reference signal to correct for the time it took the signal to travel from the speaker to the microphone.

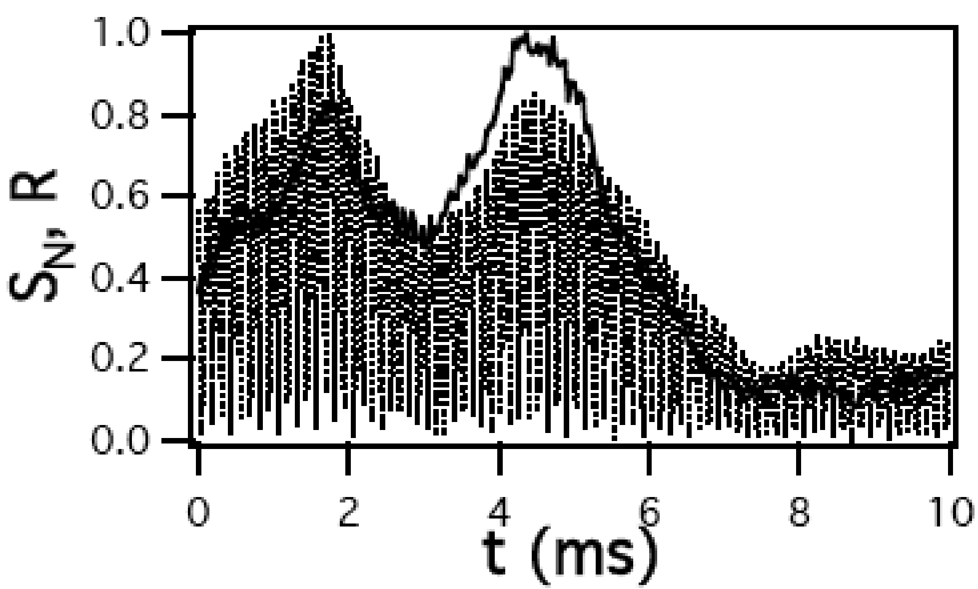

Figure 13. Normalized RKG statistic S_N (solid line) and cross correlation R (dotted line) from the experiment of Fig. 11, when the transmitted signal was a noise modulated sine wave with a center frequency of 4 kHz. Both signal were normalized so that their largest amplitude was 1.

Fig. 13 also shows the normalized RKG statistic for the reflected signal, calculated using the same reference as for the cross correlation. Both the cross correlation and the RKG statistic show peaks near 2 and 4 ms, roughly the same times as the pulse signals in Fig. 12.

Box Modes

To find acoustic modes for the box, a speaker was placed just behind the box and a microphone just in front of the box, directly in front of the hole in the face of the box. The speaker was driven with a sine wave, which was varied from 2 to 3 kHz, and the rms amplitude of the signal detected by the speaker was recorded. This test was repeated with the box absent because the amplitude of the speaker output was not constant over the range of frequencies used. The amplitude of the signal recorded by the microphone with the box present was normalized by the amplitude of a signal without the box, and the resulting amplitude is plotted in Fig. 14.

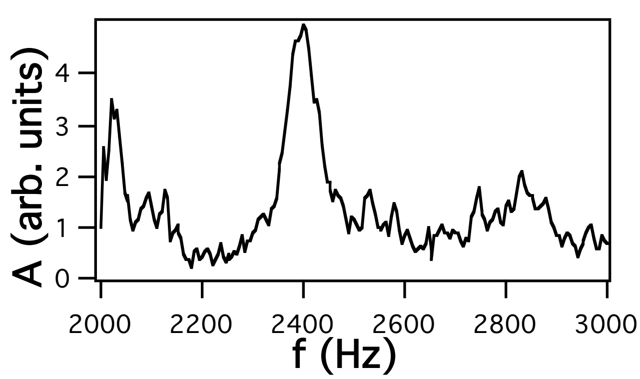

Figure 14. Response spectrum for the experimental acoustic resonator.

In Fig. 14, large modes are seen at about 2020 Hz and 2400 Hz. One would expect more large modes at frequencies below 2 kHz, but in the following experiment, it is necessary to have some spatial resolution to separate the reflection from the box from reflections off of other objects in the room. When the center frequency of the audio signal used in the experiment was below 2 kHz, the signal did not have sufficient bandwidth to resolve the box.

To find the modes when the box was located apart from the speaker, as in Fig. 11, the audio signal was a sine wave that was frequency modulated by random noise. The modulation percentage was 3%. The sine wave was loaded into an arbitrary waveform generator, and the center frequency of the sine wave could be varied by changing the output frequency of the arbitrary waveform generator.

To look for resonances using the RKG statistic, the center frequency of the audio signal was set to different values from 2 kHz to 3 kHz. The arbitrary waveform generator output a continuous signal, and from the signal received by the microphone, a 10000 point time series was digitized at 100,000 points/sec. For each different frequency, 10 of these time series were averaged to reduce noise (the ventilation system in the room was very loud).

Figure 15 shows the result of the experiment. For each center frequency, the RKG statistic was calculated for a delay value determined by the target distance, 61 cm in this case (corresponding to the peak in the RKG statistic in Fig. 13). The RKG statistic was normalized by the cross correlation between reference and received signals at the same delay value to produce S_N, the normalized RKG statistic. In order to more clearly reveal the resonances, the experiment was repeated with a non-resonant target the same size as the box, and the normalized RKG statistic S_{N0} was found for the non-resonant target. Figure 13 shows the difference between these 2 statistics.

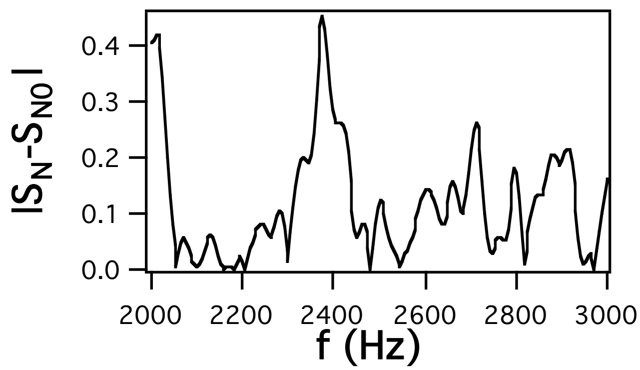

Figure 15. Absolute value of the difference between S_N , the normalized RKG statistic with the acoustic resonator present, and S_{N0}, the statistic with a non resonant target.

The plot in Fig. 15 was smoothed to reduce point-to-point scatter. The modes at 2020 and 2400 Hz are clearly visible. It is possible that some of the peaks above 2600 Hz correspond to modes with low amplitudes- it is not clear in Fig 14 if there are more modes in this range.

At first glance, the data in Fig. 15 is not too impressive. In general, in a real radar or sonar situation, we would not have a background statistic for a non-resonant target, although it might be possible to simulate such a background. I did try 2 alternate approaches to finding the mode spectrum using data from the experiment shown in Fig. 11. First, I tried a direct calculation of the phase difference between reference and received signals, with either a resonant or a non-resonant target present. The phase calculation was not able to detect the resonant modes.

Impulse Detection Method

A standard method for detecting resonance in radar is to transmit a large impulse signal and record the transient signal from the target after the end of the impulse[1]. I transmitted a 1 ms pulse of a 9 kHz signal (above 9 kHz, the speaker output dropped sharply) and recorded the transient signal from the target after the end of the pulse. The resulting signal was the same as the signal seen in Fig. 13. Ten transients were averaged, and the signal was Fourier transformed to look for frequencies corresponding to modes.

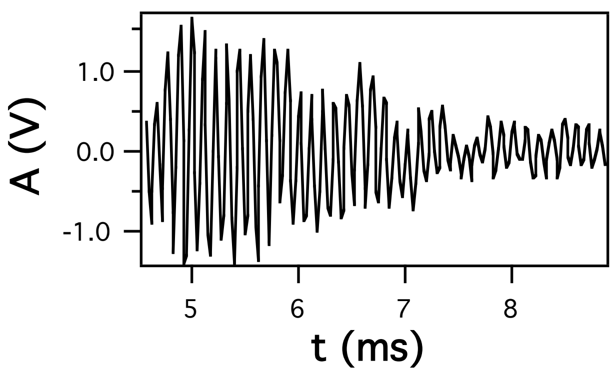

Figure 16. Portion of Fig 12(b) after main reflection from the box, expanded in scale

Figure 16 shows the transient portion of the reflected signal from the aluminum box. Fig 16 contains the same information as Fig. 12(b), but the time scale has been expanded for clarity.

In order to find information about the modes of the box from the impulse method, it is necessary to extract the mode frequencies from the transient signal in Fig. 16. The dominant frequency in Fig 16 is the 9 kHz excitation frequency, and part of this 9 kHz signal might be a reflection from the tripod supporting the box. In Fig. 12, another reflected signal is seen at about 9 ms, which limits the length of the observable transient from the box to a total of about 4 ms. The Nyquist criterion requires 2 points/cycle to resolve a frequency, so the minimum frequency resolution possible from the transient signal is 500 Hz, far too coarse to see the modes of Fig. 14. If the interfering reflection was not present, a longer transient could be recorded, but the transient amplitude is decaying in time, so the signal to noise ratio would become a problem. In contrast, the RKG statistic uses a signal of arbitrary length, so in principle arbitrary frequency resolution is possible.

The RKG statistic works better than the impulse method for several reasons. First, the RKG statistic uses a signal whose length is not limited by the length of transients or by other reflections, and the RKG method performs phase space averaging on this long signal. The RKG statistic also uses time gating, which allows the computation of the RKG statistic only for objects at a specific distance.

Conclusions

The simple statistic defined by Kaplan and Glass to detect determinism is useful for more than they intended. As I have shown here, the statistic can detect resonance using complex and even non deterministic signals, which can be useful for signature determination in radar or sonar. The statistic might also be useful for detecting phase synchronization in chaotic systems [14].

A simple experiment showed that it was possible to detect some modes of an acoustic resonator in a noisy, cluttered environment. The RKG statistic worked better than other methods for detecting resonance. With better equipment, it should be possible in the future to perform more detailed tests of this statistic.

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